

STOCHASTIC MODEL FOR A SINGLE GRADE SYSTEM WITH THREE COMPONENTS OF THRESHOLD AND CORRELATED INTER-DECISION TIMES

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ABSTRACT

It is a common phenomenon that some personnel leave an organization after completing a certain period of services to that organization voluntarily or involuntarily due to death, retirement or termination. It usually happens that whenever the policy decisions regarding promotion and target of work or sales to be achieved are revised, then, there will be exit of personnel, which in other words called the attrition or wastage. In any organization like marketing, industrial, software, the depletion of manpower due to policy decisions is quite common. This results in manpower attrition and then recruitment becomes necessary. Frequent recruitment is not advisable due to the cost of the same. Hence recruitment is postponed till a point called the breakdown point of total depletion beyond which the normal activities cannot be continued due to shortage of manpower. This level of allowable manpower attrition is called threshold. In this paper a Stochastic model to determine expected time to recruitment and variance of time to recruitment with three sources of depletion of manpower attrition under correlated inter arrival times have been derived. The Stochastic model discussed in the paper is not only applicable to industry as a whole but also in a wider context of other applicable areas.

KEYWORDS: Attrition, Threshold, Depletion of Manpower, Correlated, Inter Arrival Times

Original Article

Received: Dec 09, 2015; **Accepted:** Dec 17, 2015; **Published:** Jan 05, 2016; **Paper Id.:** IJMCARFEB20162

INTRODUCTION

Any real world situation is often considered as a mathematical model and hence the optimized solution can be obtained by using traditional techniques is the best advantage of application of random process. When the stochastic models are used in manpower planning, the profit for the organization is achieved with the formulation of appropriate policies. The policy decisions concerned with wages, targets and perquisites are the main factors for wastage in the reduction of manpower from an organization. Frequent recruitment is not recommended due to its time and expense involved. Therefore the wastage on frequent recruitment is allowed to cumulate, when the cumulative damage goes beyond the threshold level. Exits of personnel which are in other words known as threshold is an important aspect in the manpower planning.

Many models have been discussed using different kinds of wastages and different types of distributions for the threshold. Such models could be seen in [2],[3],[4],[9]and [10]. The problem of time to recruitment is

studied by several authors both for single and multi-graded systems for different types of thresholds according as the inter-decision times are independent and identically distributed random variables or correlated random variables. In a multi-graded system, transfer of personnel from one grade to another may or may not be permitted. Most of these authors have used univariate CUM policy of recruitment by which recruitment is done whenever the cumulative loss of manpower crosses a threshold. In [15] the author has obtained the performance measures namely mean and variance of the time for a two graded system when (i) the loss of manpower and the threshold for the loss of manpower in each grade are exponential random variables (ii) the inter-decision times are independent and identically distributed exponential random variables forming the same renewal process for both the grades and (iii) threshold for the organization is the max(min) of the thresholds for the two grades(max(min)model) using the above cited univariate cumulative policy of recruitment. In [1] the authors have studied the maximum model discussed in [15] when both the distributions of the thresholds have SCBZ property. In [31] the authors have obtained the performance measures when the loss of manpower follows Poisson distribution and the threshold for the loss of manpower in the two grades are geometric random variables. Assuming that the inter decision times are exchangeable and constantly correlated random variables, the performance measures of time to recruitment are obtained in [11] according as the loss of manpower and thresholds are discrete or continuous random variables. In [19] the author has extended the results in [11] for geometric thresholds when the inter-decision times for the two grades form two different renewal processes. In [32] the author has studied the results in [11] and [15] using a bivariate policy of recruitment. In [28] these performance measures are obtained when the inter-decision times are exchangeable and constantly correlated exponential random variables and the distributions of the thresholds have SCBZ property. In [30] the authors have studied the results in [15] when the threshold for the organization is the sum of the thresholds for the grades. This paper has been extended in [18] when threshold distributions have SCBZ property. In [12] the work in [30] is studied when the loss of manpower and thresholds are geometric random variables according as the inter-decision times for the two grades are correlated random variables or forming two different renewal processes. This author has also obtained the mean time for recruitment for constant combined thresholds using a univariate max policy for recruitment. In [16] the authors have studied the work in [15] when the thresholds for the loss of manpower in the two grades follow an extended exponential distribution with shape parameter 2. In [6] and [7] the authors have considered a new univariate recruitment policy involving two thresholds in which one is optional and the other a mandatory and obtained the mean time to recruitment under different conditions on the nature of the thresholds according as the inter-decision times are independent and identically distributed exponential random variables or the inter-decision times are exchangeable and constantly correlated exponential random variables. In [8] the authors have also obtained the mean time to recruitment when the optional and mandatory thresholds are geometric random variables. In [20] the authors have studied the problem of time to recruitment for a two graded manpower system when (i) the loss of manpower in the organization due to i^{th} decision is maximum of the loss of manpower in this decision in grades A and B (ii) the thresholds for the organization is max (min) of the thresholds for the loss of manpower in the two grades under different conditions using a univariate CUM policy of recruitment. They have also studied max policy of recruitment by assuming constant threshold. In [21] the authors have extended the work of [20] when the threshold for the loss of man hours in the organization is the sum of the corresponding thresholds of the two grades according as the thresholds are exponential or extended exponential thresholds. In [22], [23], [24] and [25] the authors have extended the results in [6] for a two grade system according as the thresholds are exponential random variables or extended exponential random variables or SCBZ property possessing random variables or geometric random variables. In [26] the authors have extended the results in [6] for a two grade system according as the optional thresholds are exponential random variable and the distributions of the mandatory thresholds have SCBZ property.

For a single graded manpower system, in [18] the authors have obtained the mean and variance of time to recruitment when (i) the loss of manpower form a sequence of independent and identically distributed Poisson random variables (ii) the threshold for the loss of manpower follow geometric distribution and the number of policy decisions announced by the organization is governed by a renewal process with independent and identically distributed exponential inter-decision times. In all the earlier research work the monotonicity of inter-decision times which do exists in reality, is not taken into account. In [14] the above limitation is removed and the authors have obtained the mean time to recruitment for a single grade manpower system by assuming that (i) the inter decision times form a geometric process in which the monotonicity is inbuilt in the process itself (ii) the loss of manpower is a sequence of independent and identically distributed exponential random variables and (iii) the distribution of the threshold for the loss of manpower in the organization is exponential. In [5] the authors have studied the results of [14] for a two grade system when the threshold for the loss of manpower in the two grades are exponential thresholds or SCBZ property possessing thresholds or extended exponential thresholds or geometric thresholds. They have also studied this work in [27] by considering optional and mandatory thresholds for the loss of manpower in the two grades. In [13] the authors have extended the work of [23],[24] when the loss of manpower for the organization is the maximum of the loss of manpower in the two grades by assuming exponential, extended exponential and SCBZ property possessing thresholds for the loss of manpower. In all the above cited works, authors have assumed the loss of manpower is a sequence of independent and identically distributed random variables. Uma et.al [28],[29] suggested two models in which threshold distributions following exponential and having SCBZ property to overcome the problems with the constant threshold models. A stochastic model with threshold of two components are assumed and they are (i) allowable level of wastage and (ii) available manpower also called as backup resource was developed by Vijayasankar et al [33]. The total of the maximum allowable attrition and the maximum available backup resource are considered as threshold in that paper. The backup resource is nothing but the manpower inventory on hand which can be used whenever there is a necessity. In [17], Rojamary and Uma modifies the results in [33] to find the long - run average cost per unit time for a single grade system under univariate policy of recruitment.

In the present paper, the authors extends the results of [33] to find the expected time to recruitment and the variance of time to recruitment using a bivariate policy of recruitment for a single grade system with three components of threshold namely the maximum allowable attrition and the maximum available backup resource and attrition due to transfer of personal from other organizations.

Model Description

Consider an organization taking decisions at random epochs in $[0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man-hour if a person quits. It is assumed that the loss of manhour is linear and cumulative. Let X_i be a continuous random variable representing the amount of depletion of manhours due to the i^{th} epoch of policy decisions. X_i are i.i.d random variables having exponential distribution with parameter μ . It is assumed that the inter-decision times u_i 's are constantly correlated and exchangeable random variables with probability distribution function $F(\cdot)$, mean a and R , the correlation between u_i and u_j , $i \neq j$,

$i = 1, 2, 3$. Let d be constant threshold for the number of decisions. Let $F_m(\cdot)$ be c.d.f of $\sum_{i=0}^m u_i$, the sum of inter-

decision times between decision making epochs with $F_k^*(s)$ as its Laplace transform. The loss of manpower process and the process of inter-decision times are assumed to be statistically independent. Let $Y_i \quad i = 1, 2, 3$ are the threshold levels for the loss of man-hour and are assumed to be continuous random variables following two exponential distributions and one Erlang distribution with parameters λ_1, λ_2 and λ_3 . Let T be a continuous random variable denoting the time to recruitment in the organization with probability density function $l(\cdot)$ and cumulative distribution $L(\cdot)$. From Renewal theory we note that P [there are exactly k decisions in $(0, t)$] is $F_k(t) - F_{k+1}(t)$. Let $E(T)$ be the expected time to recruitment and $V(T)$ be the variance of time to recruitment.

MAIN RESULTS

The probability distribution of T is given by

$$P(T > t) = \sum_{k=0}^d P[\text{there are exactly } k \text{ decisions}] \times P[\text{the system does not cross the threshold}]$$

$$= \sum_{k=0}^d [F_k(t) - F_{k+1}(t)] P\left[\left[\sum_{i=0}^m X_i < \max(Y_1, Y_2, Y_3)\right]\right]$$

Conditioning upon X_i and using the Law of total probability, it can be shown that

$$L(T) = P(T > t)$$

$$= 1 - [1 - [1 - g^*(\lambda_1)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_1)]^{k-1} + [1 - g(\lambda_2)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_2)]^{k-1} + [1 - g(\lambda_3)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_3)]^{k-1}]$$

$$- [1 - g(\lambda_1 + \lambda_2)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_1 + \lambda_2)]^{k-1} - [1 - g(\lambda_2 + \lambda_3)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_2 + \lambda_3)]^{k-1} -$$

$$[1 - g(\lambda_1 + \lambda_3)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_1 + \lambda_3)]^{k-1} + [1 - g(\lambda_1 + \lambda_2 + \lambda_3)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_1 + \lambda_2 + \lambda_3)]^{k-1}$$

$$- \left\{ [1 - g(\lambda_3)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_3)]^{k-1} + [1 - g(\lambda_2 - \lambda_3)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_2 - \lambda_3)]^{k-1} + \right.$$

$$= 1 - [1 - [1 - g^*(\lambda_1)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_1)]^{k-1} + [1 - g(\lambda_2)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_2)]^{k-1} + [1 - g(\lambda_3)] \sum_{k=1}^{d-1} F_k(t) [g(\lambda_3)]^{k-1}]$$

Taking Laplace Trans form, we Get

$$L^*(s) = [1 - g^*(\lambda_1)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_1)]^{k-1} + [1 - g^*(\lambda_2)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_2)]^{k-1} + [1 - g^*(\lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_3)]^{k-1}$$

$$- [1 - g^*(\lambda_1 + \lambda_2)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_1 + \lambda_2)]^{k-1} - [1 - g^*(\lambda_2 + \lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_2 + \lambda_3)]^{k-1}$$

$$\begin{aligned}
 & [1 - g^*(\lambda_1 + \lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_1 + \lambda_3)]^{k-1} + [1 - g^*(\lambda_1 + \lambda_2 + \lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_1 + \lambda_2 + \lambda_3)]^{k-1} \\
 & - \left\{ [1 - g^*(\lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_3)]^{k-1} + [1 - g^*(\lambda_2 - \lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_2 - \lambda_3)]^{k-1} \right. \\
 & \left. - [1 - g^*(\lambda_1 - \lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_1 - \lambda_3)]^{k-1} - [1 - g^*(\lambda_1 + \lambda_2 - \lambda_3)] \sum_{k=1}^{d-1} F_k^*(s) [g^*(\lambda_1 + \lambda_2 - \lambda_3)]^{k-1} \right\} \lambda_3
 \end{aligned}$$

As in Gurland, when the random variables $x_i, i = 1, 2, 3, \dots, m$ are exchangeable, exponentially distributed with constant correlation R , the c.d.f of $S_m = U_1 + U_2 + \dots + U_m$ is

$$F_k(x) = (1 - R) \sum_{i=0}^{\infty} \frac{(mR)^i}{(1 - R + mR)^{i+1}} \frac{\psi\left(m+i, \frac{x}{b}\right)}{(m+i-1)!}$$

and $F_k^*(s) = \frac{m^k}{[1 + \{kR(1-m)/(1-R)\}]}$ where $a = \frac{b}{[1-R]}$

$$\psi(n, x) = \int_0^x e^{-x} x^{n-1} dx, \quad b = c(1-R) \quad \text{and} \quad f_m^*(s) = \text{Laplace Transform of } f_m(x)$$

Assuming that X_i follow exponential distribution with parameter μ , then

$$g^*(\mu) = \frac{\mu}{\mu + \lambda}$$

On simplification, we find

$$\begin{aligned}
 E(T) = & \frac{b}{1 - R} \left[\left\{ \frac{(\mu + \lambda_1)^d - \mu^{d-1}(\mu + d\lambda_1)}{\lambda_1(\mu + \lambda_1)^{d-1}} \right\} + \left\{ \frac{(\mu + \lambda_2)^d - \mu^{d-1}(\mu + d\lambda_2)}{\lambda_2(\mu + \lambda_2)^{d-1}} \right\} \right. \\
 & \left. - \left\{ \frac{(\mu + \lambda_3)^d - \mu^{d-1}(\mu + d\lambda_3)}{\lambda_3(\mu + \lambda_3)^{d-1}} \right\} - \left\{ \frac{(\mu + \lambda_1 + \lambda_2)^d - \mu^{d-1}[\mu + d(\lambda_1 + \lambda_2)]}{(\lambda_1 + \lambda_2)(\mu + \lambda_1 + \lambda_2)^{d-1}} \right\} \right. \\
 & \left. - \left\{ \frac{(\mu + \lambda_2 + \lambda_3)^d - \mu^{d-1}[\mu + d(\lambda_2 + \lambda_3)]}{(\lambda_2 + \lambda_3)(\mu + \lambda_2 + \lambda_3)^{d-1}} \right\} - \left\{ \frac{(\mu + \lambda_1 + \lambda_3)^d - \mu^{d-1}[\mu + d(\lambda_1 + \lambda_3)]}{(\lambda_1 + \lambda_3)(\mu + \lambda_1 + \lambda_3)^{d-1}} \right\} \right. \\
 & \left. - \left\{ \frac{(\mu + \lambda_1 + \lambda_2 + \lambda_3)^d - \mu^{d-1}[\mu + d(\lambda_1 + \lambda_2 + \lambda_3)]}{(\lambda_1 + \lambda_2 + \lambda_3)(\mu + \lambda_1 + \lambda_2 + \lambda_3)^{d-1}} \right\} - \left\{ \frac{(\mu + \lambda_2 - \lambda_3)^d - \mu^{d-1}[\mu + d(\lambda_2 - \lambda_3)]}{(\lambda_2 - \lambda_3)(\mu + \lambda_2 - \lambda_3)^{d-1}} \right\} \right. \\
 & \left. + \left\{ \frac{(\mu + \lambda_1 - \lambda_3)^d - \mu^{d-1}[\mu + d(\lambda_1 - \lambda_3)]}{(\lambda_1 - \lambda_3)(\mu + \lambda_1 - \lambda_3)^{d-1}} \right\} \right]
 \end{aligned}$$

$$\left[\left\{ \frac{(\mu + \lambda_1 + \lambda_2 - \lambda_3)^d - \mu^{d-1}[\mu + d(\lambda_1 + \lambda_2 - \lambda_3)]}{(\lambda_1 + \lambda_2 - \lambda_3)(\mu + \lambda_1 + \lambda_2 - \lambda_3)^{d-1}} \right\} \lambda_3 \right] \quad (1)$$

$$\begin{aligned}
E(T^2) = & \left(\frac{b}{1-R} \right)^2 \left[\left\{ \frac{2(1+R^2)[(\mu + \lambda_1)^d - \mu^d]}{\lambda_1^2(\mu + \lambda_1)^{d-2}} \right. \right. \\
& - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_1)(\mu + \lambda_1)^{d-1} + 2R(1+R)\lambda_1^d]}{\lambda_1(\mu + \lambda_1)^{2d-2}} - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_1)^{d-1}} \left. \right\} \\
& + \left\{ \frac{2(1+R^2)[(\mu + \lambda_2)^d - \mu^d]}{\lambda_2^2(\mu + \lambda_2)^{d-2}} - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_2)(\mu + \lambda_2)^{d-1} + 2R(1+R)\lambda_2^d]}{\lambda_2(\mu + \lambda_2)^{2d-2}} \right. \\
& - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_2)^{d-1}} \left. \right\} + \left\{ \frac{2(1+R^2)[(\mu + \lambda_3)^d - \mu^d]}{\lambda_3^2(\mu + \lambda_3)^{d-2}} \right. \\
& - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_3)(\mu + \lambda_3)^{d-1} + 2R(1+R)\lambda_3^d]}{\lambda_3(\mu + \lambda_3)^{2d-2}} - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_3)^{d-1}} \left. \right\} \\
& - \left\{ \frac{2(1+R^2)[(\mu + \lambda_1 + \lambda_2)^d - \mu^d]}{(\lambda_1 + \lambda_2)^2(\mu + \lambda_1 + \lambda_2)^{d-2}} - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_1 + \lambda_2)(\mu + \lambda_1 + \lambda_2)^{d-1} + 2R(1+R)(\lambda_1 + \lambda_2)^d]}{(\lambda_1 + \lambda_2)(\mu + \lambda_1 + \lambda_2)^{2d-2}} \right. \\
& - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_1 + \lambda_2)^{d-1}} \left. \right\} - \left\{ \frac{2(1+R^2)[(\mu + \lambda_2 + \lambda_3)^d - \mu^d]}{(\lambda_2 + \lambda_3)^2(\mu + \lambda_2 + \lambda_3)^{d-2}} \right. \\
& - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_2 + \lambda_3)(\mu + \lambda_2 + \lambda_3)^{d-1} + 2R(1+R)(\lambda_2 + \lambda_3)^d]}{(\lambda_2 + \lambda_3)(\mu + \lambda_2 + \lambda_3)^{2d-2}} - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_2 + \lambda_3)^{d-1}} \left. \right\} \\
& - \left\{ \frac{2(1+R^2)[(\mu + \lambda_1 + \lambda_3)^d - \mu^d]}{(\lambda_1 + \lambda_3)^2(\mu + \lambda_1 + \lambda_3)^{d-2}} - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_1 + \lambda_3)(\mu + \lambda_1 + \lambda_3)^{d-1} + 2R(1+R)(\lambda_1 + \lambda_3)^d]}{(\lambda_1 + \lambda_3)(\mu + \lambda_1 + \lambda_3)^{2d-2}} \right. \\
& - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_1 + \lambda_3)^{d-1}} \left. \right\} + \left\{ \frac{2(1+R^2)[(\mu + \lambda_1 + \lambda_2 + \lambda_3)^d - \mu^d]}{(\lambda_1 + \lambda_2 + \lambda_3)^2(\mu + \lambda_1 + \lambda_2 + \lambda_3)^{d-2}} \right. \\
& - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_1 + \lambda_2 + \lambda_3)(\mu + \lambda_1 + \lambda_2 + \lambda_3)^{d-1} + 2R(1+R)(\lambda_1 + \lambda_2 + \lambda_3)^d]}{(\lambda_1 + \lambda_2 + \lambda_3)(\mu + \lambda_1 + \lambda_2 + \lambda_3)^{2d-2}} \\
& - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_1 + \lambda_2 + \lambda_3)^{d-1}} \left. \right\} - \left\{ \left\{ \frac{2(1+R^2)[(\mu + \lambda_3)^d - \mu^d]}{\lambda_3^2(\mu + \lambda_3)^{d-2}} \right. \right. \\
& - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_3)(\mu + \lambda_3)^{d-1} + 2R(1+R)\lambda_1^d]}{\lambda_3(\mu + \lambda_3)^{2d-2}} - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_3)^{d-1}} \left. \right\}
\end{aligned}$$

$$\begin{aligned}
 & + \left\{ \frac{2(1+R^2)[(\mu + \lambda_2 - \lambda_3)^d - \mu^d]}{(\lambda_2 - \lambda_3)^2(\mu + \lambda_2 - \lambda_3)^{d-2}} - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_2 - \lambda_3)(\mu + \lambda_2 - \lambda_3)^{d-1} + 2R(1+R)(\lambda_2 - \lambda_3)^d]}{(\lambda_2 - \lambda_3)(\mu + \lambda_2 - \lambda_3)^{2d-2}} \right. \\
 & \left. - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_2 - \lambda_3)^{d-1}} \right\} + \left\{ \frac{2(1+R^2)[(\mu + \lambda_1 - \lambda_3)^d - \mu^d]}{(\lambda_1 - \lambda_3)^2(\mu + \lambda_1 - \lambda_3)^{d-2}} \right. \\
 & \left. - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_1 - \lambda_3)(\mu + \lambda_1 - \lambda_3)^{d-1} + 2R(1+R)(\lambda_1 - \lambda_3)^d]}{(\lambda_1 - \lambda_3)(\mu + \lambda_1 - \lambda_3)^{2d-2}} - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_1 - \lambda_3)^{d-1}} \right\} \\
 & - \left\{ \frac{2(1+R^2)[(\mu + \lambda_1 + \lambda_2 - \lambda_3)^d - \mu^d]}{(\lambda_1 + \lambda_2 - \lambda_3)^2(\mu + \lambda_1 + \lambda_2 - \lambda_3)^{d-2}} \right. \\
 & \left. - \frac{d(1+R^2)\mu^{d-1}[(2\mu + \lambda_1 + \lambda_2 - \lambda_3)(\mu + \lambda_1 + \lambda_2 - \lambda_3)^{d-1} + 2R(1+R)(\lambda_1 + \lambda_2 - \lambda_3)^d]}{(\lambda_1 + \lambda_2 - \lambda_3)(\mu + \lambda_1 + \lambda_2 - \lambda_3)^{2d-2}} \right. \\
 & \left. - \frac{d\mu^{d-1}[d(1+R)^2 - 2R(1+R)]}{(\mu + \lambda_1 + \lambda_2 - \lambda_3)^{d-1}} \right\} \lambda_3 \quad (2)
 \end{aligned}$$

Using (1) and (2) we get the explicit expressions for the mean and variance of time to recruitment.

CONCLUSIONS

For real applications, the results of any research work should be viable. In the case of stochastic models this is very much essential since the results derived are based on real factors. In any industry or organization, the applications of stochastic models are of great need and it is also useful in every areas of human activity. It is important to identify those areas of human activity where the unbalanced situation arises on the demand for manpower and the supply. For the development of human resource management the transformation of real life situations into mathematical models and identification of those areas are to be analyzed. We observe that the above constructed model yields profit for the organization and also to the society.

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